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# Learning Minimal and Maximal Rules from Observations of Graph Transformations

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**Abstract:** Graph transformations have been used to model services and systems where rules describe pre and post conditions of operations changing a complex state. However, despite their intuitive nature, creating such models is a time-consuming and error-prone process. In this paper we investigate the possibility of extracting rules from observations of transformations, i.e., pairs of input and output graphs resulting from successful transformations and individual input graphs where they have failed. From such positive and negative examples, minimal rules are extracted, to be extended by context that is present in all positive examples and missing in at least one negative example. The result is a maximal and a required rule, jointly with the minimal rule defining the range of possible rules that could have created the observed transformations. We report on an implementation of the approach, evaluate its accuracy, scalability and limitations, and discuss applications to reverse engineering visual constructs from observations of object states of components under test.

**Keywords:** graph transformation, rule learning

## 1 Introduction

Reverse engineering is concerned with the extraction of specifications from existing systems. This can either be achieved by analysing, statically, the implementation of the system or by, dynamically, deriving the specification from observations of its behaviour. In the case of typed graph transformation, the specification is given by a type graph and a set of rules and the system's behaviour can be represented abstractly by a transformation relation labelled by rule names. In this setting, dynamic reverse engineering means to learn rules from observed transformations.

There are many potential applications of such a learning technique, corresponding to possible applications of graph transformations, including the learning of model transformation specifications, business process, biological and software models, etc. We are specifically interested in software models where learning means to extract interface specifications in the shape of visual contracts from components whose internal state is monitored at runtime [BMT<sup>+</sup>12]. At each state we expect to extract an object graph, which changes whenever an operation is applied to the component. Assuming that we are able to distinguish successful from failed invocations, we obtain for each operation a set of pairs of object graphs for successful invocations, and a separate

$$\begin{array}{ccccc}
 L & \xleftarrow{l} & K & \xrightarrow{r} & R \\
 m \downarrow & (1) & \downarrow & (2) & \downarrow m^* \\
 G & \xleftarrow{l^*} & D & \xrightarrow{r^*} & H
 \end{array}$$

Figure 1: DPO diagram representing a rule application

set of graphs where invocations have failed. From both we hope to derive visual contracts as interface specifications for the operation.

In this paper we concentrate on the last part of the problem, the extraction of rules from example transformations. After defining more precisely the problem, we discuss several challenges and their solutions and present an algorithm for learning rules from positive and negative examples generated by the AGG tool [AGG12]. To cope with the uncertainty of how well the set of examples observed can characterise the actual implementation of the operation, we derive not just one rule, but three: A minimal rule that characterises the effect, but with minimal precondition, a maximal rule that has as precondition the intersection of all contexts of successful transformations, and a required rule that only has context where there is evidence of its necessity. The results are evaluated and discussed, before we address related work and conclude the paper.

## 2 Problem

We assume the basic definitions of the algebraic double-pushout (DPO) approach over typed graphs [EEPT06]. A *graph*  $G = (V, E, s, t)$  consists of a set  $V$  of nodes (or vertices), a set  $E$  of edges, and source, target functions  $s, t : E \rightarrow V$ . A *type graph*  $TG$  is a distinguished graph introducing vertex and edge types. A graph morphism  $f : G \rightarrow H$  is a pair of mappings  $f_V : G_V \rightarrow H_V, f_E : G_E \rightarrow H_E$  compatible with sources and targets. An instance graph over  $TG$  is a graph  $G$  with a graph morphism  $type_G : G \rightarrow TG$ . The category of graphs typed over  $TG$  is called  $\mathbf{Graph}_{TG}$ .

A rule is a pair  $L \xleftarrow{l} K \xrightarrow{r} R$  of injective graph morphisms. In this paper we assume that  $K = L \cap R$  and  $l, r$  are inclusions. Given an instance graph  $G$ , a rule  $p$  is applied at a match  $m : L \rightarrow G$  satisfying the gluing conditions by deleting the image of  $L \setminus R$  and adding a copy of  $R \setminus L$ . That means, the application is only possible if the graph pattern  $L$  is mapped to  $G$  by  $m$  in such a way that no element in  $L$  is identified with an element in  $L \setminus R$ , and after removing the image of  $L \setminus R$  the resulting structure is still a graph, i.e., there are no dangling edges. The result of this construction is a double-pushout diagram like in Figure 1, where the pushout on the left represents deletion and the one on the right models creation. Figure 3 (A) shows a transformation rule which, when applied to the graph in the top left of (B) at the match mapping  $r$  to  $r1$ ,  $g$  to  $g1$ , and  $h$  to  $h$ , produces the graph in the top right of (B).

A graph transformation system (GTS)  $(P, \pi)$  consists of a set of rule names  $P$  and a function  $\pi$  assigning each name  $p$  a rule  $\pi(p) = L \xleftarrow{l} K \xrightarrow{r} R$ . The resulting transformation relation is denoted by  $G \xRightarrow{p} H \subseteq \mathbf{Graph}_{TG} \times \mathbf{Graph}_{TG}$ . As a running example, we consider a simple Hotel case study [HKM11] consisting of rules, such as *bookRoom*, *occupyRoom*, *checkout*, etc.

In this paper we are interested in extracting rules from example transformations. More precisely, we assume a type graph  $TG$  and set of rule names  $P$ , and for each  $p \in P$  a set  $\xrightarrow{p} \subseteq \mathbf{Graph}_{TG} \times \mathbf{Graph}_{TG}$  of successful transformations and a set of  $\not\xrightarrow{p} \subseteq \mathbf{Graph}_{TG}$  of failed transformation attempts. Our aim is to define, for each rule name  $p \in P$ , a rule  $\pi(p)$  such that  $\xrightarrow{p} \subseteq \xrightarrow{\pi(p)}$ , i.e., the positive examples are part of the rewrite relation and  $\not\xrightarrow{p} \cap \xrightarrow{\pi(p)}_1 = \emptyset$ , i.e., none of the negative examples is.<sup>1</sup>

Here we face a number of potential problems. First, the sets  $\xrightarrow{p}$  and  $\not\xrightarrow{p}$  of examples may be inconsistent, i.e., there may not be a single rule  $p$  performing all the transformations in  $\xrightarrow{p}$  and none of those in  $\not\xrightarrow{p}$ . Second, even if such a rule exists, the sets of examples may not be varied or representative enough to derive a unique result. Third, there is a tradeoff between having enough examples to form a representative set and being able to process them in reasonable time.

We address the first problem by assuming that our examples *are* consistent, generating them with the AGG tool [AGG12] for experimental purposes: Given a GTS and a start graph, rules are applied randomly to generate sets of pairs of graphs as examples to learn from. This allows us to compare the rules extracted with the original ones, as illustrated in Figure 2. Such generating graphs by AGG will simulate dynamic object extraction, by e.g. tracing a number of different sequences of system's states at runtime.

To address the problem of ambiguity, for each  $p \in P$  we will provide not a single rule, but three: a *minimal rule*  $\min(p)$ , a *maximal rule*  $\max(p)$ , and an *required rule*  $\text{req}(p)$ , all with the same effect but differing by the additional context included in the precondition. The minimal rule able to perform a given transformation can be determined using the construction proposed in [BHE09]. For the maximal rule, we add all the context that is present in all successful examples. The required rule only contains context whose necessity is confirmed by failed examples, i.e., where a failure shows that without the context present, the rule is not applicable. These three rules can assist developers to understand the range of object behaviours, with  $\text{req}(p)$  representing the best candidate to describe the functionality of the system.

To address the third problem, we have to ensure that our algorithm scales to large numbers of examples. To this end we assume that in the rules' left-hand sides there is no disconnected context, i.e., each element that is not deleted itself is there as a node being source or target to an edge that is deleted or to be created, or is connected by an undirected path to such a node or edge. Thus, elements that are deleted or created provide us with anchor points from which all other elements can be reached. Our evaluation of scalability shows that the learning effort is linear up to about 800 pairs of graphs given as examples, but exponential in the size of the graphs.

### 3 Algorithm

The example in Figure 3 illustrates our solution. Under (A) it shows the original rule *occupyRoom* which is applied randomly to generate successful pairs of graphs, shown in (B), and failed individual graphs represented in (C). The learning algorithm analyses these graphs with the purpose of discovering a rule approximating the one in (A).

<sup>1</sup> For a relation  $R \subseteq S \times S$ , by  $R_1 \subseteq S$  we denote its projection to its first component.

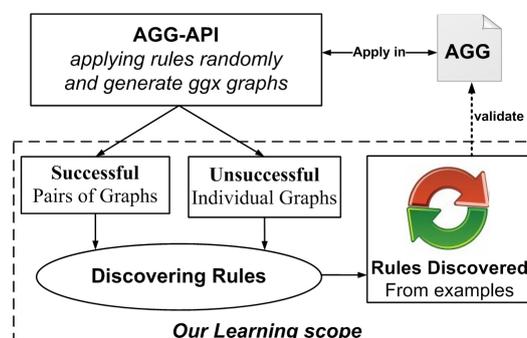


Figure 2: Learning and evaluation

The process has three stages. It begins with extracting the minimal rule for each successful pair of graphs. Intuitively, for a given pair  $(G, H)$  we know that  $L \setminus R = G \setminus H$  and  $R \setminus L = H \setminus G$ . Then,  $\min(p)$  is the smallest rule satisfying these requirements [BHE09]. In the first pair of graphs in (B), the dotted nodes and edges represent the minimal rule. It is given by  $H \setminus G$ , i.e., node  $bl:Bill$  and its outgoing edges  $bd, gi$ , as well as the nodes required to attach these edges, namely  $r1:Room$  and  $g1:Guest$ . That means, nodes in the context are part of the minimal rule if one of their incoming or outgoing edges is.

The second stage of learning is to discover the maximal rule. This requires the intersection of all successful pre-graphs, extending the minimal rule by adding to its left-hand side a representative for each graph element that occurs in all examples. Consider the two pairs of graphs given in Figure 3 (B). The matches for their minimal rules are given by nodes  $r1, g1$  and  $r7, g7$  respectively. The intersection of their contexts include the Hotel node and the second pair of room and guest present in each case, i.e.,  $r2, g2$  in the first rule and  $r6, g6$  in the second, along with their connecting edges. The maximal rule, as inferred from these two examples, is therefore isomorphic to the first example.

In the third stage, the minimal rule is extended by *required* contexts only. To prove that context is required, we need an example that fails because this context is missing.<sup>2</sup> Such an example is given under (C), where the match for the minimal rule exists, but there is no transformation because of the missing  $bi$  edge. This shows that the edge is required. The required rule is therefore the subrule of *OccupyRoom* in (A) missing only the Hotel node and adjacent edges.

The algorithm is presented more explicitly in Algorithm 1. As is mentioned above, successful examples produce the minimal and maximal rule, while the failures allow us to extend the minimal rule towards the required rule by discovering which context is necessary for the application.

Rather than operating directly on the XML output of AGG, we import all graphs into a relational database, see *line 1*, to handle efficiently large (numbers of) graphs. This is because parsing and manipulating a large number of XML instances is costly both in time and memory. A relational database provides the means to formulate complex operations on graphs as declarative

<sup>2</sup> Disregarding for now the existence of too much context in the presence of negative application conditions.

<sup>3</sup> The process at distance 5 and all nodes and edges are similar in their signatures. So which node and edge can be removed safely to get the accurate intersection?

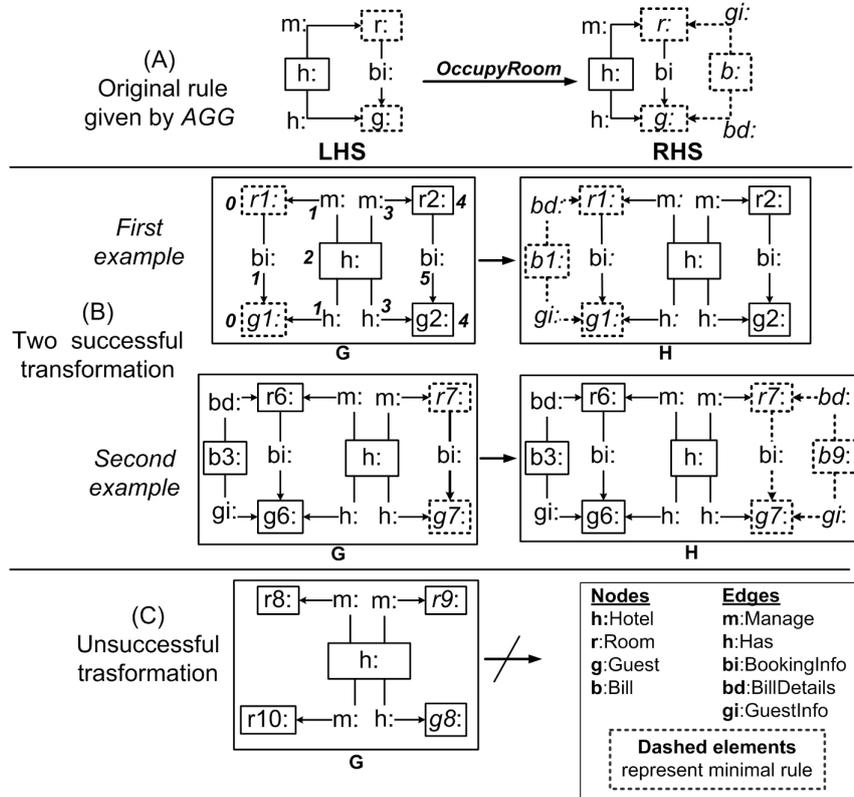
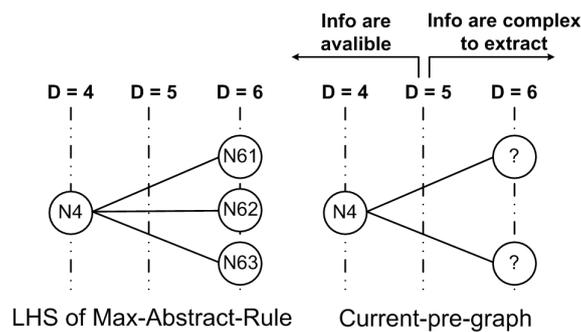


Figure 3: Original rule given by AGG Vs. successful and unsuccessful graphs


 Figure 4: Complex decision to define the accurate intersection<sup>3</sup>

queries. Similar motivations have led to the use of relational databases in graph transformation before, e.g., in [VfV06].

We initialise the computation of intersections using the smallest successful pair of graphs, called *MAR* for Maximal Abstract Rule, which is obviously an upper bound for the intersection, see line 4. We define a specific node/edge signature to make graph elements distinguishable when matching. The signature is a collection of node/edge types, their distance and abstract

ids. The distance is the shortest path to an element in the minimal rule. In Figure 3 (B) this is indicated by the numbers in the first pre-graph. The abstract id is a unique identifier in MAR.

For each successful example within the first loop in lines 7-8 we confirm that its minimal rule is isomorphic to that of MAR. If this is not the case, the transformations are not all produced by the same rule, which contradicts our assumption that the examples are consistent. Once the minimal rules are matched, the next step is to discover the intersection of their additional contexts in lines 9-12. The difficulty at this point arises when matching contexts that have similar signatures. Figure 4 describes such a situation, where it is unclear which node or edge to remove to arrive at the intersection. We overcome this problem by marking the contexts and leaving the decision to the step of updating MAR after finishing the second loop in line 13.

Another difficulty is to define the starting point for matching failed graphs. In successful examples, we start matching from the elements of the minimal rule. In failure cases, only individual graphs  $G$  are available. Therefore, we have to assume that each possible match of the left-hand side of MAR's minimal rule that satisfied the gluing conditions is a possible starting point. Then we can use the same mechanism for matching as in the successful examples, but with the purpose of discovering missing context(s), see lines 14-18.

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### Algorithm 1 Learning Graph Transformation Rules

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**Inputs:**  $SSG [G,H]$  is a set of successful pairs of graphs and  $SFG [G]$  is a set of unsuccessful individual graphs

**Outputs:** minRule, maxRule and requiredRule

---

**Begin**

```

1: initialise  $SSG$  and  $SFG$  after transferring all examples from GGX to relational DB structure.
2: discover the minRule for each  $G \Rightarrow H$  in  $SSG$ 
3: set the distances for each node(s) and edge(s)  $\in G$  in  $SSG$ 
4: let MAR Max-Abstract-Rule = the smallest pair of graphs in  $SSG$ 
5: for each  $G$  and  $H$  in  $SSG$  where  $G$  and  $H \neq$  MAR do
6:   for each node and edge  $v$  in LHS of MAR (ascending order by distances) do
7:     if  $v.distance = 0$  then
8:       confirm matching the minRule as a sub-graph isomorphism
       such that  $minG \rightarrow$  LHS of minMAR and  $minH \rightarrow$  RHS of minMAR
       Otherwise break as the rules are not consistent
9:     if  $v.distance > 0$  then
10:      discover the intersections of the contexts between  $L$  of MAR  $\cap G$ 
11:      set  $pD$  = the possibility of deleting contexts in MAR that are out of the intersection
12:      if  $pD=1$  then remove context Otherwise set  $remove++$  and mark the contexts
13:      update MAR() delete and update the context(s) that has been remarked in line 12
14: for each  $fG$  in  $SFG$  do
15:   discover all possible subsets  $\in fG$  that can match exactly the LHS of minMAR
       assume matching each subset to be the LHS of the  $min-fG$  and use the same mechanism from line 9 to 12 but
       with different conditions :
16:   if  $pD > 0$  set context.isRequired=true in MAR. As this is a missing context in  $fG$  which would give a strong
       reason that the context is required to be exists to avoid such failure.
17:   if  $pD < 0$  generate a NAC Negative Application Condition for the rule
18:   set minRule= MAR where its node(s) and edge(s) must be at  $distance=0$ 
19:   set maxRule= MAR
20:   set requiredRule=minRule +MAR where its nodes and edges are specified to be required (line 16)

```

**End**

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As a result, the relationships between minimal, maximal and required rules and the original  $\pi(p)$  is  $\min(p) \subseteq \text{req}(p) \subseteq \pi(p) \subseteq \max(p)$ . The example in Figure 3 illustrates the possibility of  $\text{req}(p) \subset \pi(p)$  as  $\text{req}(p)$  does not contain  $h:Hotel$ .

## 4 Evaluation

In this section, we apply our learning algorithm to the Hotel case study [HKM11]. The original system consists of nine rules, but we only use the subset of *bookRoom*, *freeRoom*, *occupyRoom*, *checkout* because we are not interested in conditions or computations on attributes. In Figure 5, a registered guest can book or free a room, adding or removing *bookingInfo* edges between *Room* and *Guest* nodes. If a room is booked, it can be occupied and freed by checking out.

We conduct two types of experiments, to evaluate scalability with respect to graph size and number of examples considered. Scalability is significant here because, in contrast to work on learning model transformations [Var06, BV09] our examples are not generated manually, but by observing a running implementation. That means, it may require a large number of examples to obtain enough coverage of the behaviour to allow accurate learning. We mimic this situation by generating examples using AGG.

For the first experiment, we have generated 12 examples based on graphs of increasing size and, while running the algorithm, recorded the time it takes to load them into the database, construct the minimal rule, calculate the distance of each node and edge to an element in the minimal rule, derive the maximal and required rules, and the total time. The results, reported in Table 1 and visualised in Figure 6, show that significant time is spent in loading the graphs into the database, calculating distances and minimal rules, while the more sophisticated computations of maximal and required rules are less significant. This is down to an efficient representation of graphs and rules in the database, which takes time to set up but benefits subsequent steps. Since we are planning to extend the approach towards more advanced features, such as negative application conditions or multi-objects, this is a useful observation. Nevertheless, the overall effort is exponential in the size  $n$  of the graph, very roughly  $2^{n/500-2}$  seconds.

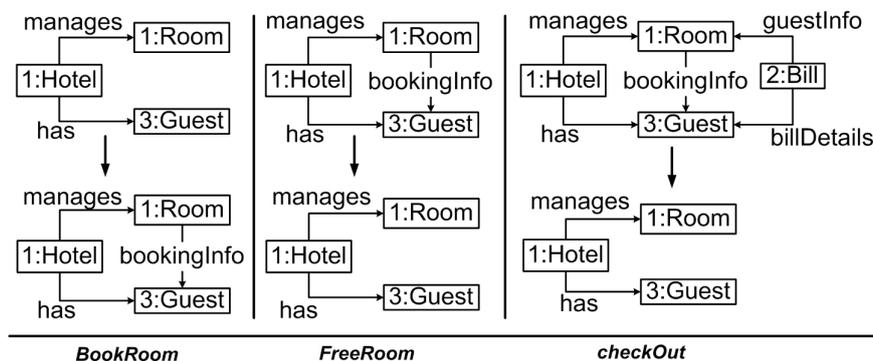


Figure 5: Main rules of Hotel system: *BookRoom*, *FreeRoom* and *Checkout* rules

The second experiment, intended to verify ability to handle significant numbers of examples, is conducted by generating, using AGG, a sequence of transformations, choosing rules and matches

No	graph size			execution time (seconds)				
	pre	post	total size	load	minimal rule	distance	max+req rule	total time
1	12	15	27	0.5650323	0.4280245	0.3270187	0.9730556	2.2931311
2	16	19	35	0.6020344	0.4260244	0.3280188	1.6990972	3.0551748
3	16	19	35	0.6030345	0.4290245	0.3270187	0.7990458	2.1581235
4	18	21	38	0.6200355	0.4240242	0.3290188	0.8210471	2.1941256
5	19	22	41	0.6450369	0.4250243	0.4090234	0.8120465	2.2911311
6	20	23	43	0.6550375	0.4310246	0.4170239	0.7980457	2.3011317
7	24	27	51	0.6850392	0.4320247	0.3280188	0.8190468	2.2641295
8	38	41	79	0.8370479	0.4310247	0.3280187	0.8070462	2.4031375
9	74	77	151	1.2210699	0.4390251	0.3380193	0.8060461	2.8041604
10	1126	1129	2255	12.228699	1.8081034	2.762158	0.9160524	17.715013
11	2074	2077	4151	22.084263	5.3053035	7.7954458	0.9640552	36.149068
12	12074	12077	24151	125.66019	157.04499	294.58885	1.7420996	579.03613
13	20074	20077	40151	204.91972	433.91382	698.92598	2.6491515	1340.4087
<b>Execution time to analyse 13 successful examples is approximately 33.251 minutes</b>								<b>1995.0731</b>

Table 1: Performance by graph size

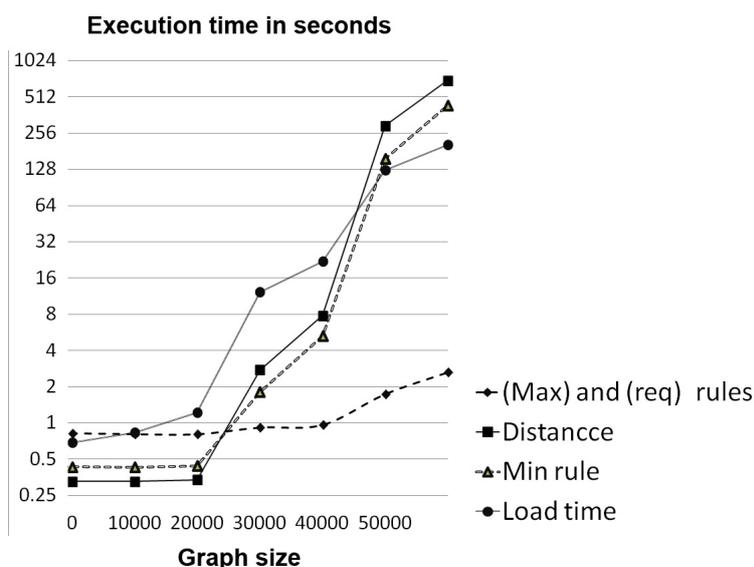


Figure 6: Performance by graph size

at random and recording for successful steps the input and output graphs and for any failed attempt at finding a match for a chosen rule, the input graph. In [Table 2](#) we report for batch sizes of 200 to 1000 graphs the time for loading, minimal rule and distance calculation as well as for construction of maximal and required rules. The actual number of examples considered is shown under *number of graphs*. For example, in the first row, out of the first 200 steps or failed attempts in the sequence, 116 where using one of the four rules of interest. The relationship between batch size and time, visualised in [Figure 7](#), is linear up to a batch size of 800, but significantly larger batches might cause problems.

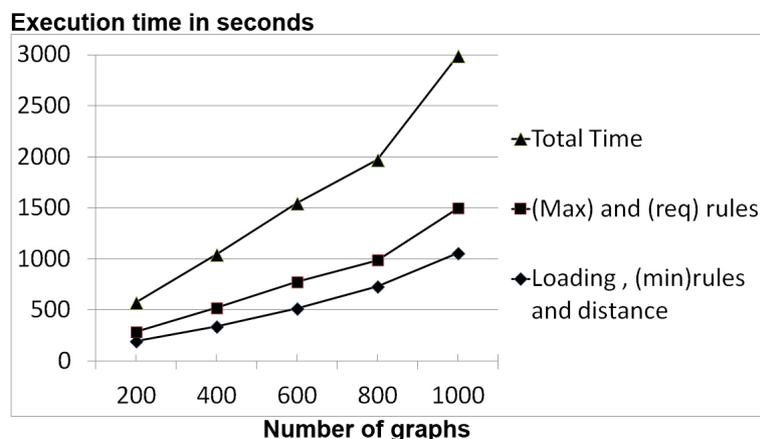


Figure 7: Performance by batch size

batch size	relevant number	loading, min rule, distance	max+req rules	total time
<b>200</b>	116	195.8182	89.9412	285.7594
<b>400</b>	238	337.6163	183.9155	521.5319
<b>600</b>	357	514.6824	257.8967	772.5791
<b>800</b>	453	729.1207	258.0947	987.2154
<b>1000</b>	565	1057.4894	438.11	1495.5995
<b>Time measured in seconds</b>				

Table 2: Performance by batch size

This is important because of the trade-off between the accuracy of the rules constructed (likely to be improved with more examples of more divers context) and the effort in processing large batches. Our finding in this experiment is that the majority of examples have very similar contexts, but a larger batch size will increase the probability of finding effective examples.

## 5 Related Work

The concept of learning rules from transformations has been suggested in a number of application areas, including the modelling of biochemical reactions [YHC09] and model transformations [DDF<sup>+</sup>11]. Although similar with respect to the overall aim of discovering rules, the challenges vary based on the nature of the graphs considered, e.g., directed, attributed or undirected graphs, the availability of typing or identity information, etc. In biochemical reactions [YHC09], the source and target graphs represent networks of biomolecules. The authors aim to discover rules modelling reactions that change the graph structure over time, based on positive examples only. They extract the minimal rule by discovering the best sub-graph match and adopt a statistical approach to rate the context. Our approach is different in that we deal with uncertainty not statistically, but by distinguishing a minimal and maximal rule and extending the minimal rule

to a required one using negative examples.

When considering approaches to learning model transformations [KLR<sup>+</sup>12], we have to distinguish two types of transformations [MV06], i.e., *in place* where source and target have the same metamodel, as in animating object diagrams by transforming them from one state to the next, and *out place*, where the metamodels are different, such as when transforming UML class diagrams to relation schemata. However, *in place* can implement *out place* by creating a joint metamodel. For learning model transformations, [DDF<sup>+</sup>11] represent input and output models as meta-model instances, supporting concepts such as attributes, inheritance, aggregation, etc. Their transformations are *out place*, so an input-output pair does not represent the result of applying a single rule, but potentially a process consisting of several steps.

[FSB12, Var06, BV09] also propose the learning of *out place* transformation rules. Therefore, they require a mapping between metamodels to specify the relation between source and target models. Being of the *in place* variety, our metamodel (type graph) is always the same and no mapping is required. [LWK10] also addresses the learning of *in place* model transformations. This approach is interactive, requiring user involvement to confirm the rules proposed by the algorithm. Our approach does not have direct user involvement and uses positive and negative examples for learning. More substantially, our application scenario is not one of a small number of carefully hand-crafted examples, but of large numbers of observations extracted from a running system. Therefore, scalability and the ability to deal with example sets providing incomplete coverage are important.

An algorithmic problem closely related to ours is graph pattern discovery. Current approaches can be classified into statistical and node signature-based solutions. Finding graph patterns by statistical means is popular in machine learning algorithms [QHJH10]. They can produce a large variance in results, depending on the frequency of a pattern. For instance, an object that is not part of the rule, but always present in the context, would be considered an important element of the rule. [QHJH10] apply decision tree learning, starting to discover matches from predefined anchor points in a hierarchical search pattern, resulting in exponential effort.

The use of node signatures can reduce this effort, but the problem remains NP-complete. [CFSV04] discusses research in exact and best graph pattern matching, most of it limited to a specific domain. A crucial point in graph or sub-graph matching is how to make nodes distinguishable when they are candidates for possible matches. For example in [JMT09], a node signature for attributed graphs is based on node/edge types and node attribute(s). Our node signatures do not include attribute(s), but distance information, which is only available due to the construction of minimal rules before matching additional context.

## 6 Conclusion

Having made first steps towards learning rules from examples of transformations, there are obvious limitations. First, our learning algorithm will not support disconnected context elements, i.e., elements not reachable from the minimal rule. This makes matching the context more efficient, but limits the usability of the approach. A generalisation is possible, but is likely to be expensive. A compromise may be to consider rule parameters and require that all context should be reachable from the minimal rule or a rule parameter. A related limitation is the handling of idle

rules, sometimes used as queries or property rules. For cases where the graph does not change during the transformation, the minimal rule is empty and thus all context is disconnected. Independently, rules with more advanced features should be supported, including negative application conditions, multi objects, attributes, etc.

Our approach is intended for reverse engineering applications in the context of model-based testing. In such a scenario it is meaningful to consider active learning, e.g., by creating additional examples in the form of test cases and observing the system's reaction, for example in order to verify the required context in case there are not enough negative examples.

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